OCR
RECOGNISING ACHIEVEMENT

Wednesday 23 January 2013 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 1 \\ 1 & 4 \end{pmatrix}$, where $a \neq \frac{1}{4}$, and **I** denotes the 2 × 2 identity matrix. Find

(i) $2\mathbf{A} - 3\mathbf{I}$, **[3]**

(ii) $\mathbf{A}^{-1}$. **[2]**

2 Find $\sum_{r=1}^{n}(r-1)(r+1)$, giving your answer in a fully factorised form. **[6]**

3 The complex number 2 – i is denoted by z.

(i) Find $|z|$ and arg z. **[2]**

(ii) Given that $az + bz^* = 4 - 8\mathrm{i}$, find the values of the real constants a and b. **[5]**

4 The quadratic equation $x^2 + x + k = 0$ has roots α and β.

(i) Use the substitution $x = 2u + 1$ to obtain a quadratic equation in u. **[2]**

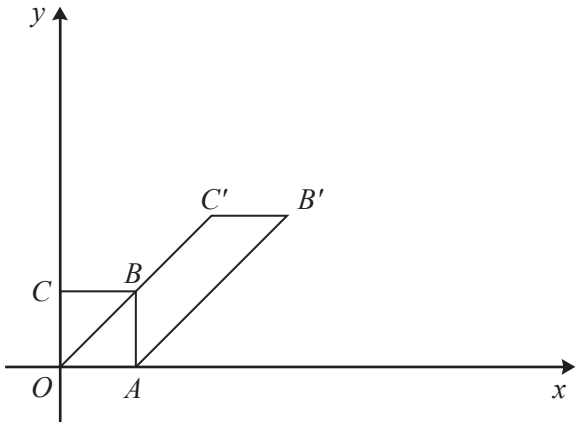
(ii) Hence, or otherwise, find the value of $\left(\frac{\alpha - 1}{2}\right)\left(\frac{\beta - 1}{2}\right)$ in terms of k. **[2]**

5 By using the determinant of an appropriate matrix, find the values of λ for which the simultaneous equations

$$\begin{aligned} 3x + 2y + 4z &= 5, \\ \lambda y + z &= 1, \\ x + \lambda y + \lambda z &= 4, \end{aligned}$$

do not have a unique solution for x, y and z. **[6]**

6



The diagram shows the unit square $OABC$, and its image $OAB'C'$ after a transformation. The points have the following coordinates: $A(1, 0)$, $B(1, 1)$, $C(0, 1)$, $B'(3, 2)$ and $C'(2, 2)$.

(i) Write down the matrix, $\mathbf{X}$, for this transformation. **[2]**

(ii) The transformation represented by $\mathbf{X}$ is equivalent to a transformation P followed by a transformation Q. Give geometrical descriptions of a pair of possible transformations P and Q and state the matrices that represent them. **[6]**

(iii) Find the matrix that represents transformation Q followed by transformation P. **[2]**

7 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z| = 2$, **[2]**

(b) $\arg(z - 3 - \mathrm{i}) = \pi$. **[3]**

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z| \leqslant 2 \text{ and } 0 \leqslant \arg(z - 3 - \mathrm{i}) \leqslant \pi. \quad \textbf{[2]}$$

8 **(i)** Show that $\dfrac{1}{r} - \dfrac{3}{r+1} + \dfrac{2}{r+2} \equiv \dfrac{2-r}{r(r+1)(r+2)}$. **[2]**

(ii) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{2-r}{r(r+1)(r+2)} = \frac{n}{(n+1)(n+2)}$. **[5]**

(iii) Find the value of $\displaystyle\sum_{r=2}^{\infty} \frac{2-r}{r(r+1)(r+2)}$. **[2]**

9 **(i)** Show that $(\alpha\beta + \beta\gamma + \gamma\alpha)^2 \equiv \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$. **[3]**

(ii) It is given that α, β and γ are the roots of the cubic equation $x^3 + px^2 - 4x + 3 = 0$, where p is a constant. Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ in terms of p. **[5]**

10 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 2$ and $u_{n+1} = \frac{u_n}{1 + u_n}$ for $n \geqslant 1$.

(i) Find u_2 and u_3, and show that $u_4 = \frac{2}{7}$. **[3]**

(ii) Hence suggest an expression for u_n. **[2]**

(iii) Use induction to prove that your answer to part **(ii)** is correct. **[5]**

Question			Answer	Marks	Guidance
1	(i)		$\begin{pmatrix} 2a-3 & 2 \\ 2 & 5 \end{pmatrix}$	B1 B1 B1 **[3]**	**I** or 3**I** seen or used 2 elements correct Other 2 elements correct
1	(ii)		$\frac{1}{4a-1}\begin{pmatrix} 4 & -1 \\ -1 & a \end{pmatrix}$ or equivalent	B1 B1 **[2]**	Divide by correct determinant Both diagonals correct
2			$\frac{1}{6}n(n+1)(2n+1)-n$ $\frac{1}{6}n(2n+5)(n-1)$	M1* DM1 A1 DM1 A2 **[6]**	Attempt to expand $(r-1)(r+1)$ Use standard result for $\sum r^2$ Obtain correct unsimplified answer Attempt to factorise Obtain completely correct answer Allow A1 if one bracket still contains a common factor
3	(i)		$\lvert z\rvert = \sqrt{5}$ $\arg z = -26.6^\circ$ or -0.464	B1 B1 **[2]**	Allow 2.2 Allow -27° or -0.46(3)
3	(ii)		$a+b=2, b-a=-8$ $a=5,\ b=-3$	B1 M1 A1 M1 A1 **[5]**	$z^* = 2 + i$ stated or used Obtain two equations from real and imaginary parts Obtain correct equations Attempt to solve 2 linear equations Obtain correct answers

Question			Answer	Marks	Guidance
4	(i)		$4u^2 + 6u + k + 2 = 0$	M1 A1 **[2]**	Substitute and attempt to simplify Obtain correct answer, must be an **equation**
4	(ii)		*Either* $\frac{k+2}{4}$ *Or* $\frac{k+2}{4}$	M1 A1ft **[2]** M1 A1	Use products of roots of new quadratic i.e. use (±) c/a Obtain correct answer, from their quadratic Use sum and product of roots of original equation Obtain correct answer
5			$3\lambda^2 - 7\lambda + 2$ $\frac{1}{3}$ or 2	M1 M1 A1 B1* DM1 A1 **[6]**	Show correct expansion process for correct 3 x 3 Correct evaluation of any 2 x 2 Obtain correct 3 term quadratic Equate their det to 0 Attempt to solve a quadratic equation Obtain correct answers
6	(i)		$\begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$	B1 B1 **[2]**	Each column correct
6	(ii)		*Either* *Or* P: $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ Q: $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$	B1 DB1 B1 B1 DB1 B1 **[6]**	*Either* *Or* Stretch, s.f. 2 in *y* direction Shear, *x*-axis invariant e.g. (0,1) → (2,1) Correct matrix Shear, *x* axis invariant e.g. (0, 1) → (1, 1) Stretch, s.f.2 in *y* direction, Correct matrix N.B. **"in the *x/y* axis" is incorrect**
6	(iii)		PQ: $\begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$ $\begin{pmatrix} 1 & 4 \\ 0 & 2 \end{pmatrix}$	M1 A1 **[2]**	Attempt at matrix multiplication of two 2 x 2 matrices from (ii) Obtain correct result cao

Question			Answer	Marks	Guidance
7	(i)	(a)		B1 B1 **[2]**	Circle Centre *O* and radius 2
7	(i)	(b)		B1 B1 B1 **[3]**	Horizontal line (3, 1) on their line ½ line to left i.e. horizontal
7	(ii)			B1 B1 **[2]**	Shade only inside their circle or above their horizontal line Completely correct diagram
8	(i)			M1 A1 **[2]**	Obtain correct numerator from addition or partial fractions Obtain **given** answer correctly
8	(ii)		$\frac{n}{(n+1)(n+2)}$	M1 A1 A1 M1 A1 **[5]**	Express at least three relevent terms using (i) 1st three terms correct Last two terms correct Show correct cancelling Obtain **given** answer correctly
8	(iii)		$-\frac{1}{6}$	M1 A1 **[2]**	Sum 1 to ∞ - 1st term or start process at $r = 2$ Obtain correct answer

Question			Answer	Marks	Guidance
9	(i)			M1 A1 A1 **[3]**	Attempt at complete expansion Obtain correct unsimplified answer Obtain **given** answer correctly
9	(ii)		*Either* $\sum\alpha = -p, \sum\alpha\beta = -4, \alpha\beta\gamma = -3$ $\frac{16-6p}{9}$ *Or* $9u^3 + (6p-16)u^2 + (8+p^2)u - 1 = 0$ $\frac{16-6p}{9}$	B1 M1 A1 M1 A1 **[5]** B1 M1 A1 M1 A1	State (anywhere) correct values for $\sum\alpha, \sum\alpha\beta, \sum\alpha\beta\gamma$ Express given expression as a single fraction Obtain correct expression using (i) Use their values for sum of roots etc. in their expression Obtain correct answer Use substitution $1/\sqrt{u}$ Rearrange appropriately and square out Obtain correct co-efficients of u^3 and u^2 Use (+/-)b/a from their cubic Obtain correct answer
10	(i)		$\frac{2}{3}, \quad \frac{2}{5}, \quad \frac{2}{7}$	B1 B1 B1 **[3]**	B1 x 3, Obtain 3 correct values Justify **given** answer
10	(ii)		$\frac{2}{2n-1}$	M1 A1 **[2]**	Fraction, in terms of *n*, with correct numerator or denominator Obtain correct answer a.e.f.
10	(iii)		$\frac{2}{2(n+1)-1}$	B1ft M1 A1 A1 B1 **[5]**	Verify result true when *n* = 1, for their (ii), or n = 2, 3 or 4 Expression for u_{n+1} using recurrence relation in terms of *n* using their (ii) Correct unsimplified answer Correct answer in correct form Specific statement of induction conclusion, previous 4 marks must be earned, *n*=1 must be verified